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A Theory and Experiments for Detecting Shock Locations

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A Theory and Experiments for Detecting Shock Locations

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ABSTRACT

In this paper we present a simplified one-dimensional theory for predicting locations of normal shocks in a converging-diverging nozzle. The theory assumes that the flow is quasi one-dimensional and the flow is accelerated in the throat area. Optical aspects of the model consider propagation of electromagnetic fields transverse to the shock front. The theory consists of an inverse problem in which from the measured intensity it reconstructs an index of refraction profile for the shock. From this profile and the Dale - Gladstone relation, the density in the flow field is determined, thus determining the shock location. Experiments show agreement with the theory. In particular the location is determined within 10 percent of accuracy. Both the theoretical as well as the experimental results are presented to validate the procedures in this work.

1. INTRODUCTION

In this paper we investigate a simplified procedure to detect the location of normal shocks in a converging diverging nozzle. The prototype model considered follows the configurations of the inlets of jet engines. The goal of the work is to locate the position of a shock generated by a flow that is depicted in figure 1. The flow is subsonic upstream and downstream. The location of the shock varies downstream of the minimum throat region. The detection process is done through an optical laser configuration (See figure 2.). The laser beam is passed through a window enclosing the throat region. The shadow of the beam is observed through a recording device as well as a sensing device (in this case a CCD array.). For theoretical purposes the patterns observed through the CCD array are described as intensity measurements of the beam.

The emphasis of this work is to interpret these measurements in a meaningful manner. In particular these measurements will be related to the theoretical values of the intensity. Once this intensity is known the theory proposes an inverse design which predicts the variation of index of refraction. The Dale- Gladstone relation [4] then yields a reconstruction of the density in the flow field. The location of a discontinuity in the density profile is interpreted as the location of the shock. The experimental technique used here is a standard shadowgraph [2]. The theory for the motion of the flow and the optics of the laser beam is a one-dimensional model. The setup for the experiment reflects these assumptions.

Previous work used for this research is as follows: The theory for the optical wave propagation follows the standard electromagnetic theory of Born and Wolf [3], the theory for the gas dynamics is based on classical one-dimensional theory found in Shapiro [1], the formulation of the inverse problem is related to the paper by Hariharan and Dunn [8], and this experimental setup is an enhancement of the work reported by Adamovsky and Eustace [5]. Additional flow visualization considerations are based on Merzkirch [2]. The form of the Dale- Gladstone formula that we used is found in [4]. Other closely related references may be found in [6], [7].

2. ELECTROMAGNETICS

As shown in figure 1, we have flow in the $\hat{x}$ direction, and thus we will have a density gradient in the $\hat{x}$ direction due to the area variation. This also causes a variation of refractive index in the $\hat{x}$ direction. Our goal is to determine the position of a normal shock that can occur downstream of the minimum throat region. We accomplish this by using transverse electromagnetic (TEM) plane waves normal to the direction of the flow (thus the $\hat{z}$ direction) and by measuring the transmission coefficient at strategic points in the field, we can reconstruct the density through out the nozzle as an inverse problem. The main difficulty we have to overcome is that the index of refraction is varying in the $\hat{x}$ direction. A one dimensional approach will allow us to analyze the situation in a simplified manner. Our idea is to consider decomposing the region into several cells spanning the throat of the nozzle treating each cell as having a constant refractive index (see figure 3). Then by sending a TEM wave in the $\hat{z}$ direction through each cell, and measuring the magnitude of either the refractive or transmission coefficients, we can solve each inverse problem using one-dimensional theory, and reconstruct our field.

Considering any cell in figure 3 as our domain, we focus on the electromagnetic theory. The incident wave is polarized as follows

$$\mathbf{E} = E(z,t)\hat{x} \tag{1}$$

$$\mathbf{H} = H(z,t)\hat{y}. \tag{2}$$

This is the configuration for a TEM wave propagating in the $\hat{z}$ direction. Both $\mathbf{E}$ and $\mathbf{H}$ are divergence free. The remaining Maxwells equations are

$$\nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t} \mathbf{H} \tag{3}$$

and

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial}{\partial t} \mathbf{E}. \tag{4}$$

Applying the assumption of constant refractive index within the cell, the incidence of a TEM wave, and the Fourier transform yields the reduced wave equations

$$\frac{\partial^2}{\partial z^2} E + k^2 n^2 E = 0 \tag{5}$$

and

$$\frac{\partial^2}{\partial z^2} H + k^2 n^2 H = 0, \tag{6}$$

where $k^2 = w^2 \mu_0 \epsilon_0$ and $\epsilon = \epsilon_0 n^2$. This problem will be solved inside the nozzle, $(0 < z < L)$, and outside the nozzle, $z < 0$ and $z > L$, with continuity conditions on the interfaces $z = 0$ and $z = L$. This system will admit solutions of the form

$$E = Ae^{-iknz} + Be^{iknz} \tag{7}$$

and

$$H = YAe^{-iknz} - YBe^{iknz} \tag{8}$$

where $Y = \sqrt{\epsilon_0/\mu_0} n$.

We define the problem in the following fashion. For $z < 0$, $n = n_\infty$ ($n_\infty = 1.00029$), and $\mathbf{E}$ is the sum of the incident and reflected wave.

$$E = e^{-ikn_\infty z} + Re^{ikn_\infty z}, \qquad z < 0, \tag{9}$$

within the cell of the nozzle

$$E = Ae^{-iknz} + Be^{iknz}, \qquad 0 < z < L, \tag{10}$$

and the transmitted wave into the air

$$E = Te^{-ikn_\infty z}, \qquad z > L. \tag{11}$$

Where,

$$A = ((1 + R) + n_\infty/n(1 - R))/2 \tag{12}$$

$$B = ((1 + R) - n_\infty/n(1 - R))/2 \tag{13}$$

$$R = \frac{2i(n + n_\infty)(n - n_\infty)sin(knL)}{(n - n_\infty)^2 e^{-iknL} - (n + n_\infty)^2 e^{iknL}} \tag{14}$$

and

$$T = \frac{(n + n_\infty)^2 - (n - n_\infty)^2}{(n + n_\infty)^2 e^{ik(n - n_\infty)L} - (n - n_\infty)^2 e^{-ik(n + n_\infty)L}} \tag{15}$$

The next observation is that the intensity is proportional to $|T|^2$, where

$$|T|^2 = \frac{16n^2n_\infty^2}{(n+n_\infty)^4 + (n-n_\infty)^4 - 2(n+n_\infty)^2(n-n_\infty)^2 cos(2knL)} \tag{16}$$

Unfortunately the wavelength of the laser demands k to be of the order 10^5 causing the intensity to be extremely sensitive to even very small changes in the refractive index, and thus rendering (16) nearly useless for the purpose of recapturing the refractive index from any given intensity distribution. Noting that near a shock the refractive index deviates only slightly from its ambient state, thus we could asymptotically consider

$$|T| = \frac{4nn_\infty}{(n_\infty + n)^2} \tag{17}$$

a fully one to one approximation of the transmission coefficient which we could use in the inversion to recover density. Applying this gave us the ability to predict a theoretical intensity distribution.

3. ONE DIMENSIONAL GAS DYNAMICS

In order to make theoretical intensity measurements we must calculate an approximation of the refractive index distribution within the nozzle. For this purpose we constructed a simple model of the flow using the classical one-dimensional gas dynamics found in [1].

The basic assumption of this model is that the flow is isentropic everywhere except at the shock position. The governing equations within the isentropic regions are

$$\left(\frac{\rho_1}{\rho_2}\right) = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = \left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{1}{\gamma-1}} \tag{18}$$

and

$$\frac{A_1}{A_2} = \frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{19}$$

where ρ is the local density, p is the pressure, A is the cross sectional area, M is the local Mach number, and $\gamma = 1.4$ for air. To bridge the solution across the shock we apply

$$M_2^2 = \frac{M_1^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}M_1^2 - 1}. \tag{20}$$

Now given the Area variation of the nozzle, and the stagnation and exit pressures we can calculate a Mach number profile within the nozzle, as shown in figure 4. Then by applying

$$\left(\frac{\rho_0}{\rho}\right)^{\gamma-1} = 1 + \frac{\gamma-1}{2}M^2 \tag{21}$$

we can calculate the corresponding density distribution, and then the Gladstone-Dale equation

$$n = n_\infty + (n_\infty - 1)(\rho/\rho_0 - 1), \tag{22}$$

where $n_\infty = 1.00029$ due to the wavelength of our laser and the room temperature, yields the refractive index of the media throughout the nozzle (See figure 5).

Now we can apply the electromagnetic theory and obtain a theoretical approximation for the relative intensity of a polarized, collimated laser passing through this media. Keeping in mind that the goal of this theory is to help analyze the intensity profile of a practical application, we performed the experiment described in the next section.

4. EXPERIMENTAL SETUP

Experimental verification of the theoretical model described in the previous section has been performed in a laboratory with a converging diverging nozzle. The nozzle is connected to a shop air supply line and has a transparent window in the test section where the shock is generated. The transparent window is approximately 25 mm in diameter and is centered at the minimum throat region of the nozzle. After passing the nozzle the air enters an expansion tank and then exits into the atmosphere. Under certain flow conditions a rapid change in the pressure occurs downstream of the minimum throat region of the nozzle. Such a rapid change in pressure is called a shock. In order to measure upstream and downstream pressure, air pressure gages are installed on the air supply line before the nozzle and on the expansion tank. A ratio of these pressure readings is related to the position of the shock inside of the nozzle. A regulator has also been installed on the expansion tank permitting control of the airflow from the tank, and thus control of the shock position.

To visualize the shock an optical shadowgraph has been used. A schematic diagram of the optical system used to generate the shadowgraph image is shown in figure 2. It consists of a single mode laser L, a pinhole PH, a lense CL, and a mirror M1. The pinhole and the collimating lense form a beam expander which generates a collimated laser beam of about 30mm in diameter. The collimated laser beam passes through the transparent test window at a right angle to the direction of the flow. To align the collimated beam with the test window the mirror M1 is equipped with angle adjusting screws and positioned on a translation stage.

A portion of the collimated beam that passes through the shock is refracted by it and generates a shadow which can be observed simply by placing photographic equipment in the path of the beam. This method of observation is a very common technique used in flow visualization [2]. This method is traditionally used for detailed analysis concerning density gradients in the flow, but here we propose to use it as a quick method of locating the shadow and predicting the shock position by using a CCD array.

5. RESULTS AND COMPARISONS

We are able to send data corresponding to the spatially varying intensity profile, as measured by the CCD array, to a data file for analysis. Because this data tends to be very noisy, we have focused on methods to filter the noise, and aide in the analysis. Noting that subsonic intensity patterns deviate little from the no flow intensity, we were led to subtract the subsonic profiles from intensity patterns under consideration. This helped remove some steady state noise, and also helped identify the shock position. To further filter the signal we performed a fast Fourier transform using the Cooley-Tukey method with 512 data points [9], and neglected modes with little contribution. The effect of this filtering is shown in figure 6.

In figure 7 we compare the theoretical data corresponding to the intensity profile with filtered experimental data for an exit vs. stagnation pressure of 0.85 for a given area variation. The theoretical data has been vertically scaled to compare with the experimental profile. The spatial scale of the experimental signal has been nondimensionalized and scaled to appropriately represent the location of the widow in the nozzle through which we send our collimated laser beam. The theoretical Mach number immediately upstream of the shock is approximately $M = 1.3$. The key feature of figure 7 is that the peak of the experimental profile and the discontinuity of the theoretical profile occur at the same normalized spatial coordinate. This implies that the location of the normal shock within the nozzle can be obtained from the peak of the experimental intensity (which corresponds to the darkest region of the shadowgraph). Inverting equation (17) and using the smaller root yields a reconstruction of the experimental refractive index field within the window of the nozzle (as shown in figure 8).

The shock positions of this experiment correspond to the theory very well for pressure ratios of 1.0 down to 0.825, but as we lower the pressure ratio below 0.825 the theoretical shock position creeps downstream of the experimental results. This is understandable because our nozzle is actually very small and by lowering the pressure ratio further, the flow becomes less and less ideal due to interaction of the shock with the boundary layer leading away from the assumptions of one dimensional gas dynamics which require isentropic flow on both sides of the shock.

6. CONCLUSIONS

By constructing a simple mathematical model of the flow of air through a nozzle and the propagation of electromagnetic waves in heterogeneous media, we have been able to analyze the signal generated by a shadowgraph with a CCD array as a measuring device, and show that the position of a normal shock within the nozzle corresponds to the position at which the signal obtains it maximum. Thus we have a model with which we can construct a sensor that can readily output the position of the shock within the illuminated region for conditions varying from no flow, to design conditions (fully supersonic outflow). Plans for future work on this topic include flow, to design conditions (fully supersonic outflow). Plans for future work on this topic include moving to larger experimental facilities, and extending the electromagnetic, and gas dynamics models to higher dimensions in order to model the experimental conditions more accurately.

7. Acknowledgment

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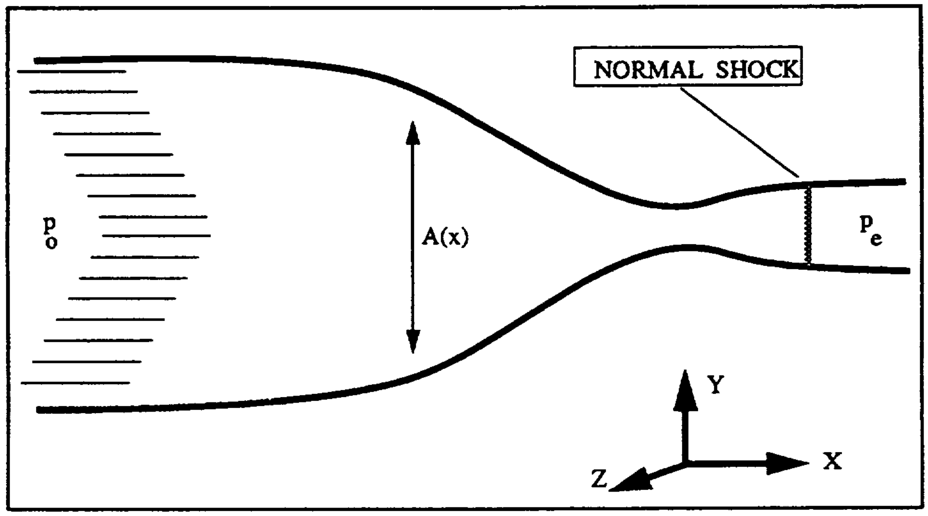

Figure 1: Nozzle flow configuration.

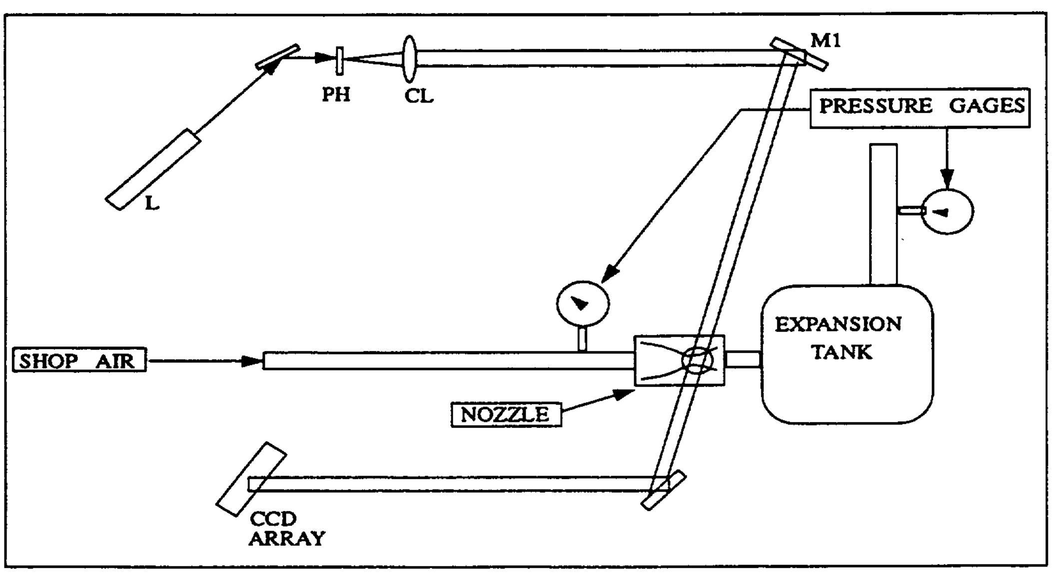

Figure 2: Configuration of experimental system.

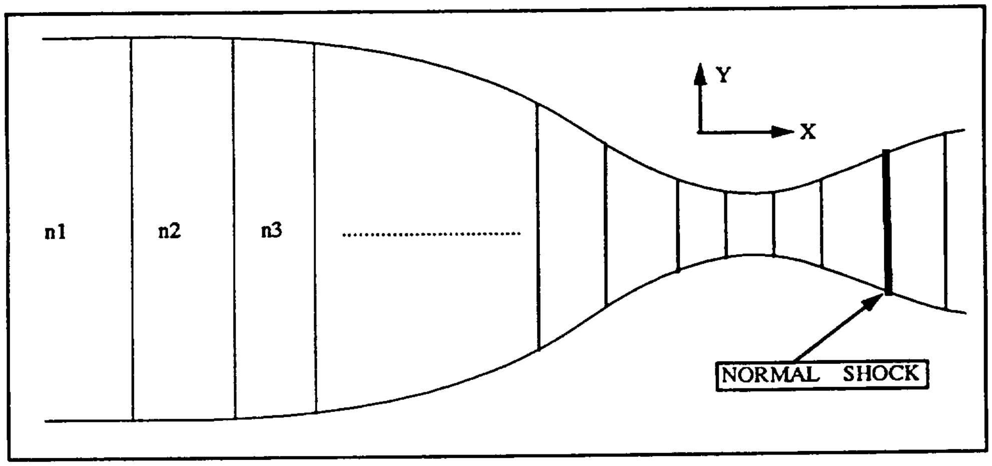

Figure 3: Discretization of the refractive index within the nozzle.

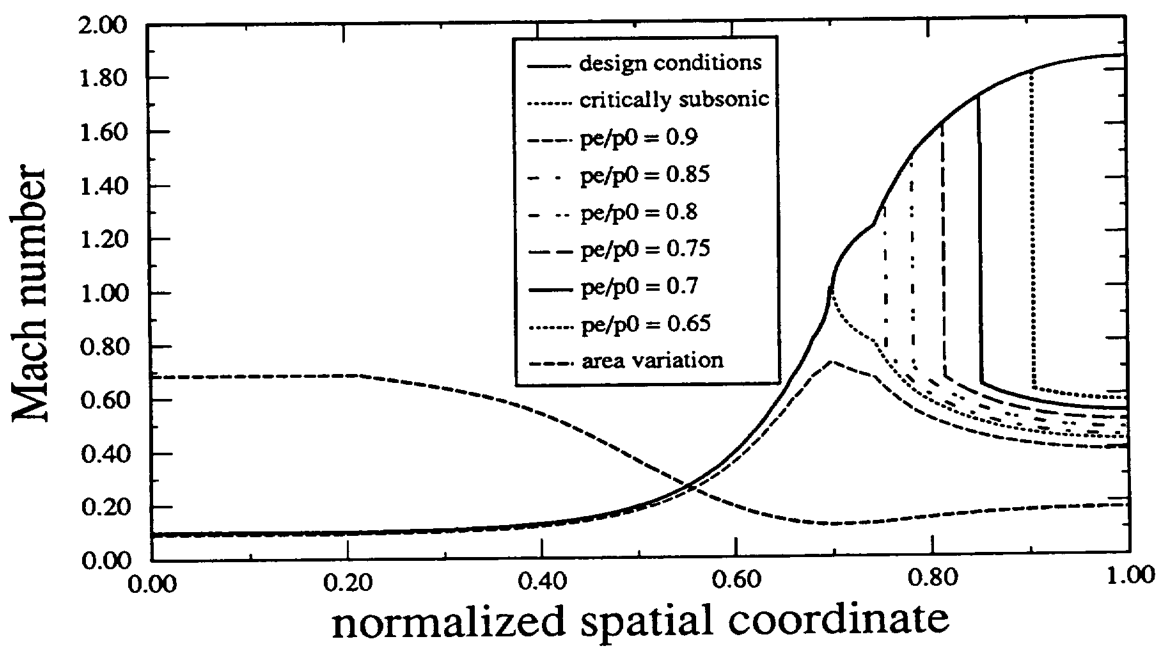

Figure 4: Theoretical Mach number profiles within nozzle.

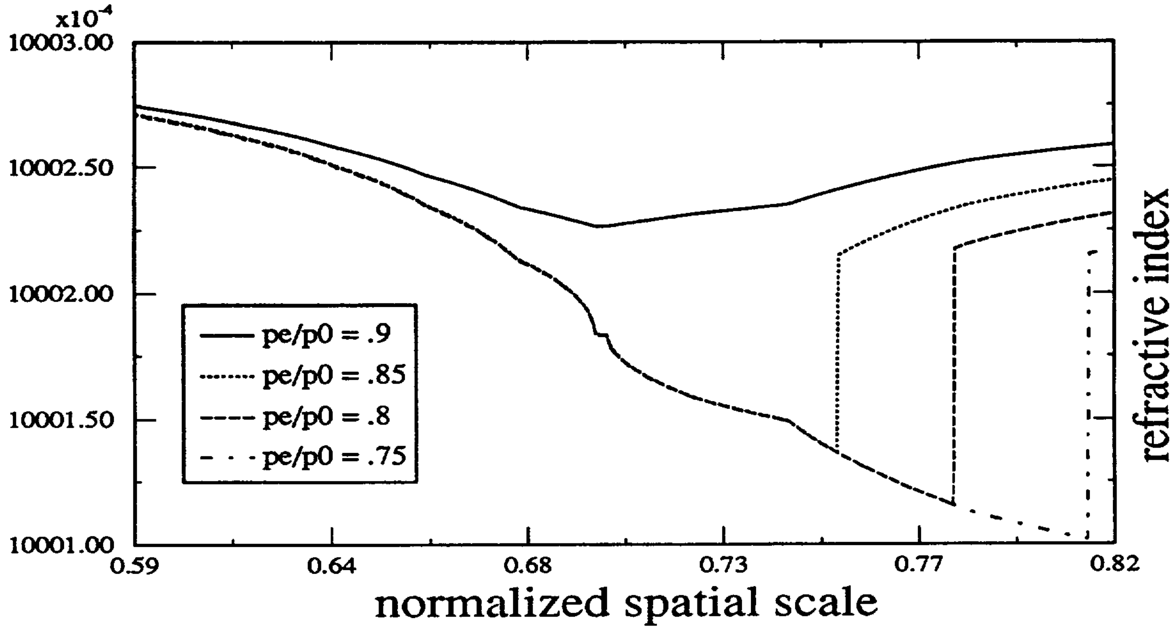

Figure 5: Theoretical refractive index in minimum throat region.

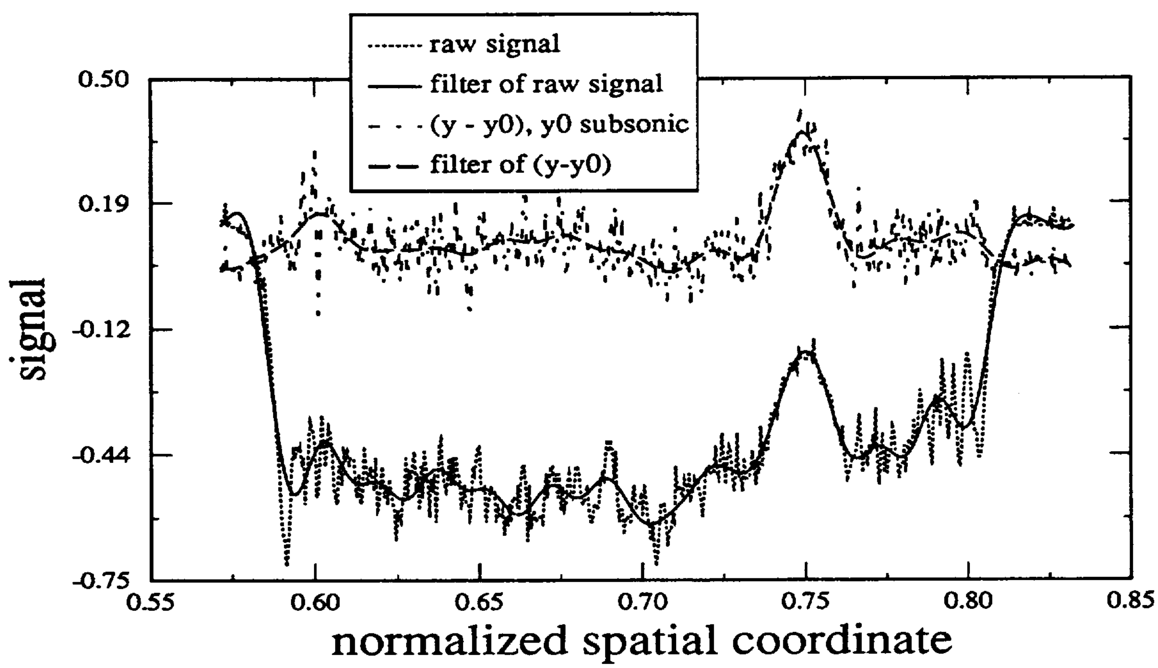

Figure 6: Experimental signals and filtered data from fast Fourier transforms.

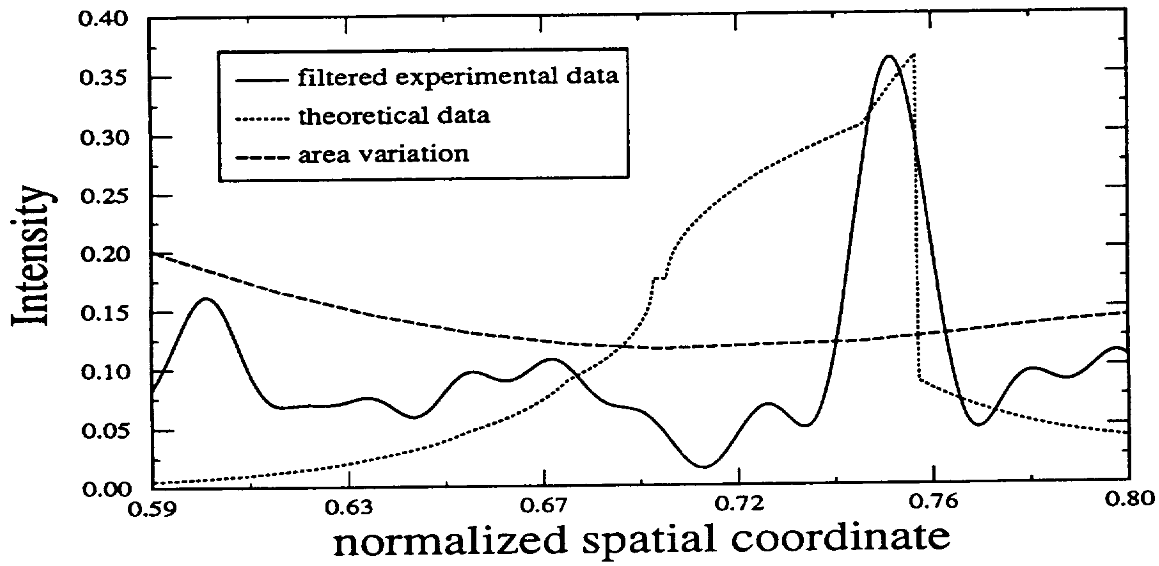

Figure 7: Comparison of theoretical data corresponding to intensity with experimental signal for pe/p0=.85 and given area.

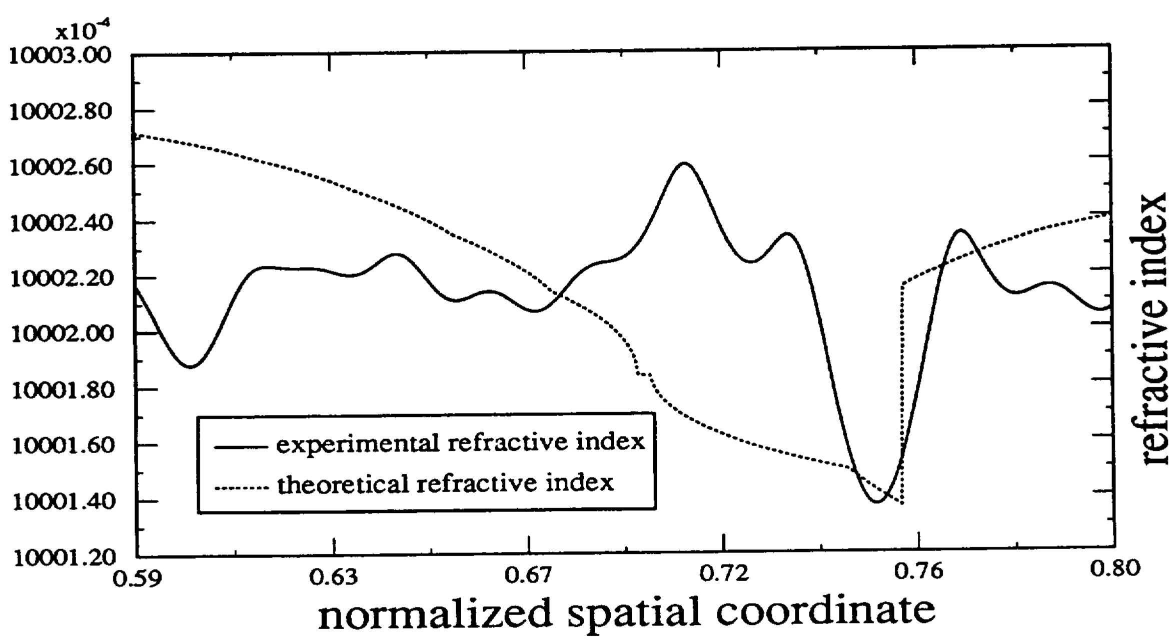

Figure 8: Comparison of predicted refractive index, and experimental approximation.

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13. ABSTRACT *(Maximum 200 words)*

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